RET/13/Test B

896	Statistics

Question Booklet No. .....

*(To be filled up by the candidate by **blue/black ball-point pen**)*

Roll No. | | | | | | | | | |

Roll No. (Write the digits in words) .....

Serial No. of OMR Answer Sheet .....

Day and Date ....

(Signature of Invigilator)

INSTRUCTIONS TO CANDIDATES

(Use only ***blue/black ball-point pen*** in the space above and on both sides of the ***Answer Sheet***)

1. Within 10 minutes of the issue of the Question Booklet, Please ensure that you have got the correct booklet and it contains all the pages in correct sequence and no page/question is missing. In case of faulty Question Booklet, bring it to the notice of the Superintendent/Invigilators immediately to obtain a fresh Question Booklet.
2. Do not bring any loose paper, written or blank, inside the Examination Hall *except the Admit Card without its envelope.*
3. *A separate Answer Sheet is given. It should not be folded or mutilated. A second Answer Sheet shall not be provided.*
4. Write your Roll Number and Serial Number of the Answer Sheet by pen in the space provided above.
5. *On the front page of the Answer Sheet, write by pen your Roll Number in the space provided at the top, and by darkening the circles at the bottom. Also, wherever applicable, write the Question Booklet Number and the Set Number in appropriate places.*
6. *No overwriting is allowed in the entries of Roll No., Question Booklet No. and Set No. (if any) on OMR sheet and Roll No. and OMR sheet No. on the Question Booklet.*
7. *Any changes in the aforesaid-entries is to be verified by the invigilator, otherwise it will be taken as unfair means.*
8. *This Booklet contains 40 multiple choice questions followed by 10 short answer questions. For each MCQ, you are to record the correct option on the Answer Sheet by darkening the appropriate circle in the corresponding row of the Answer Sheet, by pen as mentioned in the guidelines given on the first page of the Answer Sheet. For answering any five short Answer Questions use five Blank pages attached at the end of this Question Booklet.*
9. For each question, darken only one circle on the Answer Sheet. If you darken more than one circle or darken a circle partially, the answer will be treated as incorrect.
10. *Note that the answer once filled in ink cannot be changed.* If you *do not wish to attempt* a question, leave all the circles in the corresponding row blank (such question will be awarded zero marks).
11. For rough work, use the inner back page of the title cover and the blank page at the end of this Booklet.
12. Deposit ***both OMR Answer Sheet and Question Booklet*** at the end of the Test.
13. You are not permitted to leave the Examination Hall until the end of the Test.
14. If a candidate attempts to use any form of unfair means, he/she shall be liable to such punishment as the University may determine and impose on him/her.

Total No. of Printed Pages : 19

FOR ROUGH WORK

Research Entrance Test – 2013

No. of Questions : 50

Time : 2 Hours *Full Marks : 200*

Note : (i) This Question Booklet contains **40** Multiple Choice Questions followed by **10** Short Answer Questions.

(ii) Attempt as many MCQs as you can. Each MCQ carries **3 (Three)** marks. **1 (One)** mark will be deducted for each incorrect answer. Zero mark will be awarded for each unattempted question. If more than **one** alternative answers of MCQs seem to be approximate to the correct answer, choose the closest one.

(iii) Answer only 5 Short Answer Questions. Each question carries **16 (Sixteen)** marks and should be answered in **150-200** words. Blank **5 (Five)** pages attached with this booklet shall only be used for the purpose. Answer each question on separate page, after writing Question No.

1. Most of the land precipitation and evaporation on earth takes place over the :

   (1) land masses

   (2) oceans and seas

   (3) poles of the planet

   (4) subtropical latitudes

2. The downstream portion of a river :

   (1) generally becomes more sluggish

   (2) usually has turbulent flows

   (3) generally is of higher velocity, which is marked by reduced turbulence

   (4) has lower discharges than do upstream portions

3. Which of the following is not a fatty acid ?

   (1) Stearic acid

   (2) Palmitic acid

   (3) Oleic acid

   (4) Phenyl acetic acid

4. Which of the following compounds is not an antibiotic ?

   (1) Penicillin

   (2) Chloramine-T

   (3) Streptomycin

   (4) Chloramphenicol

5. The acceleration with which a particle moves in a straight line, according to the law $v^2 = 4a(x \sin x + \cos x)$, v being the velocity of the particle at a distance x from a fixed point, is :

   (1) 0

   (2) $2ax \cos x$

   (3) $4ax \cos x$

   (4) $2ax \sin x$

6. If $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} A \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the matrix A is :

(1) $\begin{bmatrix} 3 & -4 \\ 3/4 & -1 \end{bmatrix}$

(2) $\begin{bmatrix} -13/4 & 3/2 \\ 5/4 & -1/2 \end{bmatrix}$

(3) $\begin{bmatrix} -17/4 & 3/4 \\ -7/4 & -1/4 \end{bmatrix}$

(4) $\begin{bmatrix} 5/4 & 11/4 \\ 3 & -9/4 \end{bmatrix}$

7. If the error in the measurement of radius of sphere is 0.3%, then the percentage error in the measurement of its volume is :

(1) 0.15%

(2) 0.6%

(3) 0.9%

(4) 0.03%

8. The resistance of series combination of two resistances is S. When they are joined in parallel, the total resistance is P. If S = nP, then the minimum possible value of n is :

(1) 3

(2) 4

(3) 2.1

(4) 0.89

9. Mitochondria are associated with the function of :

(1) cellular digestion

(2) circulation

(3) protein synthesis

(4) cellular respiration

10. In which parts of eyes, rods and cones are present ?

(1) Retina

(2) Iris

(3) Cornea

(4) Lens

11. Let $X_1, X_2, \ldots, X_n$ be a random sample from a Bernoulli distribution with parameter $p; 0 < p < 1$. For estimating p, the bias of the estimator $\left(\sqrt{n} + 2\sum_{i=1}^{n} X_i\right) \Big/ \left(2n + 2\sqrt{n}\right)$ is :

(1) $\frac{1}{\sqrt{n}+1}\left(\frac{1}{2} + \frac{p}{\sqrt{n}}\right) - p$

(2) $\frac{1}{\sqrt{n}+1}\left(p - \frac{1}{2}\right)$

(3) $\frac{1}{\sqrt{n}+n}\left(\frac{1}{2} - p\right)$

(4) $\frac{1}{\sqrt{n}+1}\left(\frac{1}{2} - p\right)$

12. Given the ANOVA table :

Source	d.f.	SS	MSS	F
Replication	2	0.0971		
Treatments	4	0.7324	A	4.19
Error	8	B		
Total	14	1.1790		

The values of (A, B) are :

(1) 0.1831, 0.3495 (2) 0.1731, 0.3395 (3) 0.1732, 0.3385 (4) 0.1631, 0.3285

13. Let $X_1, X_2, \ldots, X_N$ be a random sample from $N_p(\mu, \Sigma)$ and $A = \sum_{i=1}^{N} (X_i - \bar{X})(X_i - \bar{X})'$. The MLE of Σ is :

(1) $\frac{1}{N-1}A$ (2) $\frac{1}{N}A$ (3) A (4) $\frac{N}{N-1}A$

14. A random variable X takes values $-2, -1, 0, 1, 2$ with probabilities $\frac{1}{6}, \frac{1}{6}, \frac{1}{3}, \frac{1}{6}, \frac{1}{6}$ respectively. Then :

(1) $E|X| > \sum_{n=1}^{2} P[|X| \geq n]$

(2) $E|X| \geq 1 + \sum_{n=1}^{2} P[|X| \geq n]$

(3) $E|X| < \sum_{n=1}^{2} P[|X| \geq n]$

(4) $E|X| = \sum_{n=1}^{2} P[|X| \geq n]$

15. Let $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)}$ be the order statistics of a random sample of size n from the uniform distribution $\cup(-\theta, \theta)$. Then statistic $T = (X_{(1)}, X_{(n)})$ is :

(1) complete but not sufficient

(2) sufficient but not complete

(3) both sufficient and complete

(4) neither sufficient nor complete

16. In a 2^3 factorial experiment $[abc] - [bc] + [ac] - [c] - [ab] + [b] - [a] + 1$ denotes the total of effect :

(1) C (2) AC (3) BC (4) AB

17. In a Markov chain with state space $\{0, 1, 2\}$ and one-step transition matrix given by

$$P = \begin{bmatrix} 1/4 & 3/4 & 0 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/4 & 3/4 \end{bmatrix}$$

the value of $p_{01}^{(2)}$ will be :

(1) 3/4 (2) 9/16 (3) 3/16 (4) 7/16

18. Suppose that the random variable X, representing the life time of a unit, follows exponential distribution with mean σ. Suppose that the MLE of σ, based on a sample of size 4 is 2.0. The MLE of the reliability function of the unit at mission time $X = 4$ is :

(1) $e^{-1/2}$ (2) $e^{-1/8}$ (3) e^{-2} (4) e^{-8}

19. In stratified sampling with population size equal to 1000, the population is divided into two strata with sizes $N_1 = 600$ and $N_2 = 400$ respectively. Under Neyman allocation if $S_1 = 24$ and the sample sizes from two strata are in the ratio $n_1 : n_2 :: 2 : 1$, then the value of S_2 is :

(1) 16 (2) 36 (3) 18 (4) 24

20. For testing the null hypothesis $H_0 : \theta = \frac{1}{2}$ against the alternative hypothesi $H_1 : \theta = \frac{3}{4}$ on the basis of a single observation X from the distribution $b(2, \theta)$, test function is defined as

$$\begin{aligned} \phi(x) &= 1 \quad \text{if} \quad x > 2 \\ &= \frac{1}{20} \quad \text{if} \quad x = 2 \\ &= 0 \quad \text{if} \quad x < 2 \end{aligned}$$

The size of the test is :

(1) $\frac{1}{80}$ (2) $\frac{1}{20}$ (3) $\frac{1}{10}$ (4) $\frac{1}{60}$

21. When the population multiple correlation coefficient is zero and R is the sample multiple correlation coefficient of a sample of size n from a p-variate normal distribution then the distribution of $\frac{(N-p)R^2}{(p-1)(1-R^2)}$ is :

(1) F with $(p-1)$ and $(N-p)$ degrees of freedom

(2) Beta distribution of first kind

(3) Two parameter gamma distribution

(4) Non-central chi-square distribution

22. Given failured censored sample $x_1 < x_2 < \ldots < x_r$ from a life testing experiment when the total item put on the test were n. If X follows exponential distribution with mean λ, then the MLE of λ is :

(1) $\frac{1}{n}\sum_{i=1}^{n} x_i$ (2) $\frac{1}{r}\left[\sum_{i=1}^{r} x_i + (n-r)x_r\right]$

(3) $\frac{1}{r}\sum_{i=1}^{r} x_i$ (4) $\frac{1}{n}\left[\sum_{i=1}^{r} x_i + (n-r)x_r\right]$

23. On the life table with $l_x = (100 - x)/190$, $5 \le x \le 100$, the probability of dying within 5 years for a man aged 40 (that is, ${}_5q_{40}$) will be :

(1) 0.286 (2) 0.421 (3) 0.083 (4) 0.118

24. In a BIBD with t treatments in b blocks of K plots each and r replicates, which one of the following is not *true* ?

(1) $rt = bk$ (2) $b \ge t$ (3) $r \ge k$ (4) $b \le (r + t - k)$

25. Let $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)}$ be the order statistic of a random sample from $U(\theta, \theta+1)$, $\theta \in \mathbb{R}$ and $T_1 = X_{(n)}$, $T_2 = \frac{1}{2}\left[X_{(1)} + X_{(n)}\right]$ and $T_3 = T_2 - \frac{1}{2}$. Then which one of the following statement is *true* ?

(1) Both T_1 and T_2 are MLE of θ but T_3 is not

(2) T_1 is MLE of θ but T_2 and T_3 are not

(3) T_3 is MLE of θ but T_1 and T_2 are not

(4) Both T_1 and T_3 are MLE of θ but T_2 is not

26. A necessary and sufficient condition for the existence of a feasible solution to a transportation problem with $a_i (i = 1, 2, \ldots, m)$ as the supply from the i^{th} origin and $b_j (j = 1, 2, \ldots, n)$ as the demand of the j^{th} destination is that :

(1) $\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$ (2) $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$ (3) $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$ (4) $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

27. Which one of the following statement is correct for Midzuno sampling scheme ?

(1) Out of the selected n units from a population of size N, all the units are selected with probability proportional to size scheme

(2) The first unit is selected with probability proportional to size scheme and rest of the $(n-1)$ units of the sample are selected out of remaining $(N-1)$ units with simple random sampling without replacement scheme

(3) All the n units of the sample are selected with equal probability of selection method

(4) The ratio estimator becomes a biased estimator under this scheme

28. In usual notations the optimum cost under optimum allocation in stratified random sampling is :

(1) $\left(\sum W_i C_i\right)\left(\sum W_i S_i^2\right) \Big/ \left(V_o + \frac{1}{N}\sum W_i S_i^2\right)$

(2) $\left(\sum W_i S_i\right)\left(\sum W_i S_i / C_i\right) / V_o$

(3) $\left(\sum W_i S_i\right)\left(\sum W_i S_i \sqrt{C_i}\right) / V_o$

(4) $\left(\sum W_i S_i \sqrt{C_i}\right)^2 \Big/ \left(V_o + \frac{1}{N}\sum W_i S_i^2\right)$

29. For the following linear programming problem :

Maximize $Z = 3x_1 + 2x_2$

subject to $x_1 + 2x_2 + s_1 = 6$

$2x_1 + x_2 + s_2 = 8$

$-x_1 + x_2 + s_3 = 1$

$x_2 + s_4 = 2$

$x_1, x_2, s_1, s_2, s_3, s_4 \geq 0$

the optimum table was obtained as

i	Basis	Cost	3	2	0	0	0	0
			x_1	x_2	s_1	s_2	s_3	s_4
1	x_2	2	0	1	2/3	-1/3	0	0
2	x_1	3	1	0	-1/3	2/3	0	0
3	s_3	0	0	0	-1	1	1	0
4	s_4	0	0	0	-2/3	1/3	0	1

The optimum solution of the problem will be :

(1) $x_2 = 4/3,\ x_1 = 10/3$

(2) $x_1 = 4/3,\ x_2 = 10/3$

(3) $x_1 = 38/3,\ x_2 = 0$

(4) $x_1 = 0,\ x_2 = 38/3$

30. Let $N(t)$ follows a Poisson process and $s < t$. Then the probability $P[N(s) = k \mid N(t) = n]$ will be :

(1) $e^{-s/t}\dfrac{(s/t)^k}{\lfloor k}$

(2) $\binom{n}{k}\left(\frac{t}{s}\right)^n\left\{1-\frac{t}{s}\right\}^{n-k}$

(3) $\binom{n}{k}\left(\frac{s}{t}\right)^k\left\{1-\frac{s}{t}\right\}^{n-k}$

(4) $\left(\frac{s}{t}\right)^k\left(1-\frac{s}{t}\right)^{n-k}$

31. You conduct a hypothesis test and you observe values for the sample mean and sample standard deviation when $n = 25$ that do not lead to the rejection of H_0. You calculate a p-value of 0.0667. What will happen to the p-value if you observe the same sample mean and standard deviation for a sample of size greater than 25 ?

(1) Increase

(2) Decrease

(3) Remain the same

(4) Nothing can be said exactly

32. Let $x_1, x_2, \ldots, x_n$ be a random sample from a $N(0, \theta)$ distribution where the variance θ is unknown. The UMP test for $H_0 : \theta = \theta_0 (> 0)$ against $H_1 : \theta > \theta_0$ is of the form :

(1) $\sum_{i=1}^{n} x_i^2 = C$, where C is some constant

(2) $\sum_{i=1}^{n} x_i \geq C$, where C is some constant

(3) $\sum_{i=1}^{n} x_i \leq C$, where C is some constant

(4) $\sum_{i=1}^{n} x_i^2 \geq C$, where C is some constant

33. Bayes estimator is one :

(1) Which minimizes the posterior expected loss for each given set of observations

(2) Which maximizes the posterior expected loss for each given set of observations

(3) Which maximizes the likelihood function for a given prior

(4) Which attains the minimum mean square error

34. For testing a simple hypothesis against a simple alternative, Bayes factor becomes nothing but the :

(1) Weighted likelihood ratio

(2) Likelihood ratio

(3) Likelihood ratio for only normal posteriors

(4) An infinite quantity

35. A bridge system has 4 paths and 4 cuts. The system will survive till :

(1) Any one cut survives

(2) Any one path survives

(3) Either any one cut or any one path survives

(4) All are true

36. For a negative exponential distribution

$$f(t) = \lambda e^{-\lambda t},\ 0 < \lambda < \infty,\ 0 \le t < \infty$$

intensity function $i(t)$ and hazard function $h(t)$:

(1) $i(t) \neq h(t)$ (2) $i(t) > h(t)$ (3) $i(t) = h(t)$ (4) $i(t) < h(t)$

37. Let X_1, X_2 and X_3 be independent random variables with $X_k (k = 1, 2, 3)$ having p.d.f.

$$f_k(x) = k\theta e^{-k\theta x}\ ;\ 0 < x < \infty,\ \theta > 0$$

Then a sufficient statistic for θ is :

(1) $X_1 + X_2 + X_3$ (2) $X_1 + 2X_2 + 3X_3$

(3) $X_1 X_2 X_3$ (4) $3X_1 + 2X_2 + X_1$

38. Let $X_1, X_2, \ldots, X_n$ be a random sample from Bernoulli distribution with parameter p and $S = \sum_{i=1}^{n} X_i$. Then MLE of $P[S = 1]$ is :

(1) $S(1-S)^{n-1}$ (2) $\frac{S}{n}\left(1 - \frac{S}{n}\right)^{n-1}$ (3) $S\left(1 - \frac{S}{n}\right)^{n-1}$ (4) $\left(\frac{S}{n}\right)\left(1 - \frac{S}{n}\right)$

39. In a population, the following logistic model was used to estimate the size of the population at time t

$$P(t) = \frac{1/A}{1 + \frac{B}{A}e^{-ut}},\ t \ge 0$$

Given that $P(0) = 46687 \times 10^3$ and $P(\infty) = 245 \times 10^6$ the value of A would be :

(1) $\frac{1}{46687 \times 10^3}$ (2) $\frac{1}{245 \times 10^6}$ (3) zero (4) 4.248

40. Which one of the following possesses dual recording system of vital events ?

(1) Census (2) National Sample Survey

(3) National Family Health Survey (4) Sample Registration System

Attempt any five questions. Write answer in 150-200 words. Each question carries 16 marks. Answer each question on separate page, after writing Question Number.

1. The joint probability density function of X and Y is $f(x, y) = \frac{1}{2x^2 y}$ for $1 \le x < \infty$ and $\frac{1}{x} < y < x$. Find the conditional distribution of $X \mid Y$.

2. Explain Lahiri's method for drawing a PPSWR sample.

3. What is conjugate analysis in Bayesian inference ?

4. Write a brief note on Rao-Blackwell versus Lehman Scheffe theorem.

5. Given a failured censored sample $x_1 < x_2 < \ldots < x_r$ from exponential distribution when the number of items put to test were n and failed items were not replaced. Obtain the MLE of mean life and reliability function.

6. State and prove a necessary and sufficient condition for the existence of a feasible solution to a transportation problem.

7. The distribution function of a random variable X is :

$$F_X(x) = 0 \ , \ x < -2$$
$$= \frac{1}{2} \ , \ -2 \le x < 2$$
$$= 1 \ , \ x \ge 2$$

Find the characteristic function of X.

8. Explain the meanings and concepts of main effects and interaction effects in design of experiments. Write these effects for a 2^3 experiment.

9. Let $\{X_n, n > 0\}$ be a Markov chain with three states $\{0, 1, 2\}$ and with transition matrix

$$\begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$$

The initial probability distribution is

$$P[X_0 = i] = 1/3, \ i = 0, 1, 2$$

Find

(i) $P[X_2 = 2, X_1 = 1 | X_0 = 2]$

(ii) $P[X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2]$

10. In the context of life table, explain the notations : l_x, ${}_5q_x$, ${}_5d_x$, T_x and e_x^o and mention how these are calculated.

Roll No. : .....

Q. No. :

Roll No. : .....

Q. No. :

Roll No. : .....

Q. No. :

Roll No. : .....

Q. No. :

Roll No. : .....

Q. No. :

FOR ROUGH WORK