RET/14/Test B		895	Mathematics

Question Booklet No. ....

(To be filled up by the candidate by **blue/black ball-point pen**)

Roll No | | | | | | | | | |

Roll No. (Write the digits in words) ....

Serial No. of OMR Answer Sheet ....

Day and Date ....

(Signature of Invigilator)

INSTRUCTIONS TO CANDIDATES

Use only ***blue/black ball-point pen*** in the space above and on both sides of the ***Answer Sheet***)

1. Within 10 minutes of the issue of the Question Booklet, Please ensure that you have got the correct booklet and it contains all the pages in correct sequence and no page/question is missing. In case of faulty Question Booklet, bring it to the notice of the Superintendent/Invigilators immediately to obtain a fresh Question Booklet.
2. Do not bring any loose paper, written or blank, inside the Examination Hall *except the Admit Card without its envelope.*
3. *A separate Answer Sheet is given. It should not be folded or mutilated. A second Answer Sheet shall not be provided.*
4. Write your Roll Number and Serial Number of the Answer Sheet by pen in the space provided above.
5. *On the front page of the Answer Sheet, write by pen your Roll Number in the space provided at the top, and by darkening the circles at the bottom. Also, wherever applicable, write the Question Booklet Number and the Set Number in appropriate places.*
6. *No overwriting is allowed in the entries of Roll No., Question Booklet No. and Set No. (if any) on OMR sheet and Roll No. and OMR sheet No. on the Question Booklet.*
7. *Any changes in the aforesaid-entries is to be verified by the invigilator, otherwise it will be taken as unfair means.*
8. *This Booklet contains 40 multiple choice questions followed by 10 short answer questions. For each MCQ, you are to record the correct option on the Answer Sheet by darkening the appropriate circle in the corresponding row of the Answer Sheet, by pen as mentioned in the guidelines given on the first page of the Answer Sheet. For answering any five short Answer Questions use five Blank pages attached at the end of this Question Booklet.*
9. For each question, darken only one circle on the Answer Sheet. If you darken more than one circle or darken a circle partially, the answer will be treated as incorrect.
10. *Note that the answer once filled in ink cannot be changed.* If you *do not wish to attempt* a question, leave all the circles in the corresponding row blank (such question will be awarded zero marks).
11. For rough work, use the inner back page of the title cover and the blank page at the end of this Booklet.
12. Deposit *both OMR Answer Sheet and Question Booklet* at the end of the Test.
13. You are not permitted to leave the Examination Hall until the end of the Test.
14. If a candidate attempts to use any form of unfair means, he/she shall be liable to such punishment as the University may determine and impose on him/her.

Total No. of Printed Pages : 15

FOR ROUGH WORK

Research Entrance Test – 2014

No. of Questions : 50

Time : **[illegible] Hours** | *Full Marks : 200*

Note : (i) This Question Booklet contains **40** Multiple Choice Questions followed by **10** Short Answer Questions.

(ii) Attempt as many MCQs as you can. Each MCQ carries **3 (Three)** marks. **1 (One)** mark will be deducted for each incorrect answer. Zero mark will be awarded for each unattempted question. If more than **one** alternative answers of MCQs seem to be approximate to the correct answer, choose the closest one.

(iii) Answer only 5 Short Answer Questions. Each question carries **16 (Sixteen)** marks and should be answered in **150-200** words. Blank **5 (Five)** pages attached with this booklet shall only be used for the purpose. Answer each question on separate page, after writing Question No.

1. Which of the following is ***not*** a greenhouse gas ?
(1) Carbon dioxide (2) Methane (3) Sulphur dioxide (4) Nitrogen

2. The saliva of mammals contains starch splitting enzyme. The name of that enzyme is :
(1) Amylase (Ptyalin) (2) Secretin (3) Lysozyme (4) Mucin

3. Cytosine in DNA combines with :
(1) Adinosine (2) Uracil (3) Guanine (4) Thiamine

4. If Vectors $2i-j+k,\ i+2j-3k,\ 3i+\lambda j+5k$ are coplanar, then the value of λ is :
(1) –2 (2) –3 (3) –4 (4) –5

5. The value of $\left(-1+i\sqrt{3}\right)^{3/2}$ is :
(1) $\sqrt{2}$ (2) $2\sqrt{2}$ (3) $2+\sqrt{2}$ (4) $2-\sqrt{2}$

6. The number of electrons contained in 1 Coulomb of charge equals to :
(1) 6.25×10^{17} (2) 6.25×10^{18} (3) 6.25×10^{19} (4) 1.6×10^{19}

7. A unit mass of solid is converted to liquid at its melting ; the heat required for this process is the :
(1) Specific heat (2) Latent heat of vaporization
(3) Latent heat of fusion (4) External latent heat

8. Granite is :
(1) a sedimentary rock (2) a metamorphic rock
(3) a volcanic rock (4) a plutonic igneous rock

9. Coal is a :
(1) Sedimentary rock (2) Hydrothermal deposit
(3) Low-grade metamorphic rock (4) High-grade metamorphic rock

10. Which one of the following gases is present in the stratosphere that filters out some of the sun's ultraviolet light and provides an effective shield against radiation damage to living things ?
(1) Oxygen (2) Methane (3) Ozone (4) Helium

11. A cube is set rotating under no forces about its centre with uniform angular velocity. After certain time,
(1) only angular velocity will be changed
(2) only angular momentum will be changed
(3) both angular velocity and angular momentum will be changed
(4) neither angular velocity nor angular momentum will be changed

12. The geometrical equations of a rigid body having n generalized co-ordinates do not contain time variable explicitly, then :
(1) Hamilton function will be H = T – V
(2) Hamilton function will be H = T + V
(3) Lagrange function will be L = T + V
(4) Hamilton characteristic function will be A = 2T, where T and V are kinetic and potential energy respectively.

13. If p_r, q_r are momentum variables and generalized co-ordinates of a system, then :
(1) $p_r = \frac{\partial H}{\partial q_r}$, H is Hamilton function
(2) $p_r = \frac{\partial T}{\partial q_r}$, T is kinetic energy
(3) $p_r = \frac{\partial S}{\partial q_r}$, S is Hamilton principal function
(4) $p_r = \frac{\partial L}{\partial q_r}$, L is Lagrange function

14. The components of velocity of an incompressible fluid in the case of a two-dimensional flow at the point (x, y) are (ax, – ay), where a is a constant. The equation of the stream line passing through the point (2, 2) is :
(1) xy = 1 (2) xy = 2 (3) xy = 3 (4) xy = 4

15. The image system for a source outside a circle consists of :
(1) an equal source at the inverse point and an equal source at the centre of the circle
(2) an equal source at the inverse point and an equal sink at the centre of the circle
(3) an equal sink at the inverse point and an equal source at the centre of the circle
(4) an equal sink at the inverse point and an equal sink at the centre of the circle

16. If σ_1, σ_2 and σ_3 are principal stresses at a point, then the first stress invariant is :
(1) $\sigma_1 + \sigma_2 + \sigma_3$
(2) $\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1$
(3) $\sigma_1 \sigma_2 \sigma_3$
(4) $\sigma_1 \sigma_2^2 + \sigma_2 \sigma_3^2 + \sigma_3 \sigma_1^2$

17. If a function $f : \mathbb{C} \longrightarrow \mathbb{C}$ is defined by :
$f(z) = f(x + iy) = u(x, y) + iv(x, y) = x \ \forall z \in \mathbb{C}$,
where $\mathbb{C}$ is the set of complex numbers, then :
(1) $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial y}$ do not exist at z = 0
(2) Cauchy - Riemann equations are satisfied but f is not differentiable at z = 0
(3) f is differentiable at z = 0
(4) Cauchy - Riemann equations are not satisfied for any value of z

18. For the power series $f(z)=\sum_{n=0} a_n z^n, \frac{1}{R}=\overline{\lim_{n\to\infty}} |a_n|^{\frac{1}{n}}, R\neq 1,$ then :

(1) the radius of convergence of this series is R^2

(2) f(z) has no singularity on $|z|=R$

(3) f(z) has at least one singularity on $|z| = R$

(4) f(z) has at least one singularity on $|z| = \frac{R}{2}$

19. Let $f(x)=x, g(x)=x^2 \forall x \in [0,1]$

If $\int_0^1 f\,dg = \mu(g(1)-g(0))$ then the value of μ is :

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) $\frac{3}{2}$ (4) $\frac{1}{2}$

20. A function $f:[0,1]\to \mathbb{R}$ is defined by :

$$f(x)=\begin{cases}1, & \text{when } x \text{ is irrational in } [0,1]\\ 0, & \text{when } x \text{ is rational in } [0,1]\end{cases}$$

Then f is :

(1) Riemann as well as Lebesgue integrable over [0, 1]

(2) Lebesgue integrable but not Riemann integrable over [0, 1]

(3) Neither Riemann nor Lebesgue integrable over [0, 1]

(4) Riemann integrable and Riemann integral of f over [0, 1] is 1.

21. Let (X, s, μ) be a measure space and f be an extended real-valued measurable function on X such that $\int_X f\,d\mu$ exists. Define V_f on s by :

$$V_f(E)=\int_E f\,d\mu \;\forall E \in s$$

Then V_f is :

(1) measure as well as signed measure on X

(2) measure but not signed measure on X

(3) signed measure but not measure on X

(4) neither measure nor signed measure on X

22. The series :

$(1-x)^2 + x(1-x)^2 + x^2(1-x)^2 + \dots\dots\dots\dots, \forall x \in [0,1]$ is :

(1) point-wise as well as uniformly convergent in [0, 1]

(2) not point-wise convergent in [0,1]

(3) point-wise but not uniformly convergent in [0,1]

(4) point-wise as well as uniformly convergent in $\left[\frac{1}{2},1\right]$

23. Let G be a group of order p^n, p = prime integer, n = positive integer, then :
(1) O (Z(G)) < 1 (2) O (Z(G)) = 1 (3) O (Z(G)) > 1 (4) O (Z(G)) = p^n

24. Tell which is the ***correct*** statement for the following groups :
(1) $\mathbb{Z}_2 \times \mathbb{Z}_2 \cong \mathbb{Z}_4$ (2) $\mathbb{Z} \times \mathbb{Z}$ is cyclic (3) $\mathbb{Z}_2 \times \mathbb{Z}_3 \cong \mathbb{Z}_6$ (4) $\mathbb{Z}_2 \times \mathbb{Z}_6 \cong \mathbb{Z}_{12}$

25. Non abelian groups of order 6 are :
(1) 1 (2) 3 (3) 2 (4) $\lfloor 6$

26. Total number of composition series of the group $\mathbb{Z}_{30}$ are :
(1) 4 (2) 5 (3) 8 (4) 6

27. Let Q be the field of rational numbers. Then the degree of Q $(\sqrt{3})$ over Q $(\sqrt{2})$ is :
(1) 4 (2) 10 (3) 6 (4) 2

28. If E_1 and E_2 are the splitting fields of the polynomials x^2+3 and x^2+x+1 over the field of rationales Q, then :
(1) $E_1 \not\subset E_2$ (2) $E_2 \not\subset E_1$ (3) $E_1 = E_2$ (4) $E_1 \neq E_2$

29. A C^2 function u satisfying $\nabla^2 u = 0$ is called a :
(1) Poisson function (2) Gauss function
(3) Green's function (4) Harmonic function

30. If $\Phi(x,t)$ is a fundamental solution of heat equation, then which of the following is ***true*** ?
(1) $\int_{R^n} \Phi(x,t)dx = 0$ (2) $\int_{R^n} \Phi(x,t)dx = 1$ (3) $\int_{R^n} \Phi(x,t)dx = e^x$ (4) $\int_{R^n} \Phi(x,t)dx = \log x$

31. Let a function $f: R^2 \to R$ is given by $f(x) = x_1^2 + x_2^2$, then its Hessian matrix is given by :
(1) $\begin{bmatrix} 0 & 2 \\ 1 & -2 \end{bmatrix}$ (2) $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ (3) $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$ (4) $\begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$

32. Which of the following points satisfy first order necessary condition for the optimization problem ?
Minimize $x_1^2 + \frac{1}{2}x_2^2 + 3x_2$
subject to $x_1, x_2 > 0$
(1) $[0 \ \ 3]^T$ (2) $[1 \ \ 2]^T$ (3) $[1 \ \ 1]^T$ (4) $[0 \ \ 0]^T$

33. Let d be feasible direction of $f: R^n \to R$ then directional derivative $\frac{\partial}{\partial d} f(x)$ at x is defined as :
(1) $\lim_{t \to 0} \left[\frac{f(x+td) - f(x)}{t}\right]$ (2) $\lim_{t \to 0} \left[\frac{f(x-td) - f(x)}{t}\right]$
(3) $\lim_{d \to 0} \left[\frac{f(x+td) - f(x)}{d}\right]$ (4) $\lim_{d \to 0} \left[\frac{f(x-td) - f(x)}{d}\right]$

34. If primal of a linear programming problem is :
Minimize $C^T x$
subject to $Ax = b, x \geq 0$
then its dual is given by :
(1) max $\lambda^T b$, subject to $\lambda^T A \leq C^T$.
(2) min $\lambda^T b$, subject to $\lambda^T A = C^T$.
(3) max $\lambda^T b$, subject to $\lambda^T A = C^T$.
(4) min $\lambda^T b$, subject to $\lambda^T A \geq C^T$.

35. Euler's equations of motion of a rigid body are used for :
(1) rotation under finite forces about frame of reference fixed in the space
(2) rotation under finite forces about the frame of reference fixed with the body
(3) linear motion under impulsive forces
(4) linear motion under finite forces

36. The radius of convergence of the power series $\sum_{n=1}^{\infty} n^{n^2} z^{n^3}$ is :
(1) 1
(2) e
(3) $\frac{1}{e}$
(4) e^2

37. The solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$, where $z = f(x)$ and $\frac{\partial z}{\partial y} = g(x)$ on y = 0, is given by :
(1) $z(x,y) = \frac{1}{2}(f(x-y) + f(x+y)) + \frac{1}{2}\int_{x-y}^{x+y} g(u)\,du$
(2) $z(x,y) = \frac{1}{2}(f(x-y) + f(x+y)) + \frac{1}{2}\int_{x}^{x+y} g(u)\,du$
(3) $z(x,y) = \frac{1}{2} f(x-y) + \frac{1}{2}\int_{x-y}^{x+y} g(u)\,du$
(4) $z(x,y) = \frac{1}{2}(f(x-y) + f(x+y)) + \frac{1}{2}\int_{y}^{x+y} g(u)\,du$

38. If the series $\sum_{n=0}^{\infty} C_n$ is convergent and $f(x) = \sum_{n=0}^{\infty} C_n x^n, x \in (-1, 1)$ then $\lim_{x \to 1} f(x)$ is :
(1) $1 + \sum_{n=0}^{\infty} C_n^2$
(2) $\sum_{n=0}^{\infty} C_n$
(3) $2 + \sum_{n=0}^{\infty} C_n$
(4) $3 + \sum_{n=0}^{\infty} C_n^3$

39. A set expressed as the intersection of a finite number of half spaces is called a :
(1) polyhedron
(2) non-convex polytope
(3) convex polytope
(4) concave polytope

40. The number of cyclic subgroups of order 10 of the group $\mathbb{Z}_{100} \times \mathbb{Z}_{25}$ are :
(1) 10
(2) 14
(3) 20
(4) 24

Attempt any five questions. Write answer in 150-200 words. Each question carries 16 marks. Answer each question on separate page, after writing Question Number.

1. State and prove conservation law of energy using Lagrangian approach.

2. Find the equation of continuity in the spherical polar co-ordinates.

3. Prove that all possible norms defined on a finite dimensional vector space X over K (= $\mathbb{R}$ or $\mathbb{C}$) are equivalent.

4. Let (X, ζ) be a topological space and B be a sub collection of ζ. Prove that B is a base of ζ if and only if every ζ-open set is expressed as a union of members of B.

5. Let G be a group and let O(G) = pq, where p, q are distinct primes, p < q and $p \times (q-1)$. Show that G is cyclic.

6. Let $f(x) \in F[x]$ be of degree $n \geq 1$. Then show that there is a finite extension E of F of degree atmost $\underline{|n}$ in which $f(x)$ has n roots.

7. Show that two dimensional Laplace equation $\nabla^2 u = 0$ in polar co-ordinates takes the form :

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

8. Consider the conjugate direction algorithm to find minimizer of :

$f(x_1, x_2) = \frac{1}{2} x^T \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix} x - x^T \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ with initial point $x^0 = [0\ 0]^T$ and a conjugate direction $d^0 = [1, 0]^T$, then find the minimizer at one iterate.

9. If a function $f : [a, b] \to \mathbb{R}^n, n \geq 1,$ is continuous on [a, b] and differentiable on (a, b) then prove that there exists c ∈ (a, b) such that :

$$\| f(b) - f(a) \| \leq (b-a) \| f'(c) \|$$

10. If (X, || ||) is a normal linear space over a field K (= $\mathbb{R}$ or $\mathbb{C}$) and x be a non-zero vector in X then prove that there is a bounded linear functional F on X such that :

$$F(x) = \|x\| \text{ and } \|F\| = 1$$

Roll No. : ....

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FOR ROUGH WORK