Science

RET/15/Test B

895	Mathematics

39

Question Booklet No. 26

(*To be filled up by the candidate by* **blue/black ball-point pen**)

Roll No. | | | | | | | | | |

Roll No. (Write the digits in words) ....

Serial No. of OMR Answer Sheet ....

Day and Date ....

(Signature of Invigilator)

INSTRUCTIONS TO CANDIDATES

(Use only ***blue/black ball-point pen*** in the space above and on both sides of the ***Answer Sheet***)

1. Within 10 minutes of the issue of the Question Booklet, Please ensure that you have got the correct booklet and it contains all the pages in correct sequence and no page/question is missing. In case of faulty Question Booklet, bring it to the notice of the Superintendent/Invigilators immediately to obtain a fresh Question Booklet.
2. Do not bring any loose paper, written or blank, inside the Examination Hall *except the Admit Card without its envelope.*
3. *A separate Answer Sheet is given. It should not be folded or mutilated. A second Answer Sheet shall not be provided.*
4. Write your Roll Number and Serial Number of the Answer Sheet by pen in the space provided above.
5. *On the front page of the Answer Sheet, write by pen your Roll Number in the space provided at the top, and by darkening the circles at the bottom. Also, wherever applicable, write the Question Booklet Number and the Set Number in appropriate places.*
6. *No overwriting is allowed in the entries of Roll No., Question Booklet No. and Set No. (if any) on OMR sheet and Roll No. and OMR sheet No. on the Question Booklet.*
7. *Any changes in the aforesaid-entries is to be verified by the invigilator, otherwise it will be taken as unfair means.*
8. *This Booklet contains 40 multiple choice questions followed by 10 short answer questions. For each MCQ, you are to record the correct option on the Answer Sheet by darkening the appropriate circle in the corresponding row of the Answer Sheet, by pen as mentioned in the guidelines given on the first page of the Answer Sheet. For answering any five short Answer Questions use five Blank pages attached at the end of this Question Booklet.*
9. For each question, darken only one circle on the Answer Sheet. If you darken more than one circle or darken a circle partially, the answer will be treated as incorrect.
10. *Note that the answer once filled in ink cannot be changed.* If you *do not wish to attempt* a question, leave all the circles in the corresponding row blank (such question will be awarded zero marks).
11. For rough work, use the inner back page of the title cover and the blank page at the end of this Booklet.
12. Deposit *both OMR Answer Sheet and Question Booklet* at the end of the Test.
13. You are not permitted to leave the Examination Hall until the end of the Test.
14. If a candidate attempts to use any form of unfair means, he/she shall be liable to such punishment as the University may determine and impose on him/her.

SEAL

Total No. of Printed Pages : 19

FOR ROUGH WORK

Research Entrance Test – 2015

No. of Questions : 50

Time : 2 Hours *Full Marks : 200*

Note : (i) This Question Booklet contains **40** Multiple Choice Questions followed by **10** Short Answer Questions.

(ii) Attempt as many MCQs as you can. Each MCQ carries **3 (Three)** marks. **1 (One)** mark will be deducted for each incorrect answer. **Zero** mark will be awarded for each unattempted question. If more than **one** alternative answers of MCQs seem to be approximate to the correct answer, choose the closest one.

(iii) Answer only **5** Short Answer Questions. Each question carries **16 (Sixteen)** marks and should be answered in **150-200** words. Blank **5 (Five)** pages attached with this booklet shall only be used for the purpose. Answer each question on separate page, after writing Question No.

1. Neoprene is polymer of :

   (1) Orlon (2) SAN (3) ABS (4) All of these

2. The reagent that can be used to distinguish between Glucose and Fructose is :

   (1) Bromine water (2) Fehling's solution

   (3) Tollen's reagent (4) Phenyl hydrazine

3. What will happen if a lysosome leaks inside the cell ?

   (1) The lysosomal enzymes will digest cell organelles

   (2) The lysosomal enzymes will become nonfunctional at pH 7.4 of the cytoplasm

   (3) The lysosomal enzymes will be secreted out of the cell

   (4) The leaked suicidal bag will make cell to commit suicide

4. Oxygen evolved during photosynthesis in plants comes from :

   (1) Splitting of water molecules

   (2) Breakdown of carbon dioxide

   (3) Carbohydrates accumulated by plants

   (4) Lipids

5. The contribution of Gregor Johann Mendel is related to the area of :

   (1) Plant classification (2) Genetics

   (3) Cell structure (4) Plant functions

6. Himalaya is :

   (1) Paleozoic tectonic mountain (2) Recent Folded mountain

   (3) Indian mountain (4) Eurasian mountain

7. A particle executes simple harmonic motion under the restoring force provided by a spring. The time period is T. If the spring is dived in two equal parts and one part is used to continue the simple harmonic motion, the time period will :

(1) remain T (2) become 2T (3) become T/2 (4) become $T/\sqrt{2}$

8. The efficiency of the Carnot's engine working between the steam point and the ice point is :

(1) 36.81% (2) 26.81% (3) 40% (4) 16.8%

9. If $\vec{a} = 2i - 3j + 4k$ and $\vec{b} = 3i + 2j$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(1) 45° (2) 90° (3) 180° (4) 120°

10. The value of the integral $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ is

(1) π (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{4}$ (4) $-\frac{\pi}{4}$

11. A rod is set rotating about its one end in space in any manner. Then :

(1) The number of Euler's angle and degree of freedom both will be two.

(2) The number of Euler's angle will be three and degree of freedom will be two.

(3) The number of Euler's angle and degree of freedom will be three.

(4) The number of Euler's angle will be one and degree of freedom will be infinite.

12. If kinetic energy and potential energy of a system are given by $T = \frac{1}{2}\left(\dot{q}_1^2 + \dot{q}_2^2 + q_1\dot{q}_2 + 6\dot{q}_1\right)$ and $V = C + \frac{1}{2}q_1$, then :

(1) q_1 and q_2 are cyclic coordinates

(2) q_1 is cyclic coordinate

(3) q_2 is cyclic coordinate

(4) Neither q_1 nor q_2 is cyclic coordinate

13. For the expressions of kinetic energy, $T = \frac{1}{2}\left(\dot{\theta}^2 + \dot{\phi}^2 + \dot{\theta}\,\dot{\phi}\right)$ and potential energy $V = \frac{1}{2}(\theta + \phi) + C$, Hamiltonian function may be written as :

(1) H = T − V (2) H = T + V (3) H = T/V (4) H = TV

14. The flow formed by the velocity vector $\overline{q} = (-ay, ax, 0)$, where a is a constant, is :

(1) not a possible flow

(2) a possible rotation flow

(3) an irrotational flow

(4) a possible irrotational flow

15. The Reynolds number is ratio of :

(1) inertia force to viscous force

(2) inertia force to gravity force

(3) viscous force to thermal force

(4) inertia force to thermal force

16. The boundary value problem corresponding to the integral equation $y(x) = \lambda \int_0^x (x - t)\, y(t)\, dt - \lambda x \int_0^1 (1 - t)\, y(t)\, dt$ is :

(1) $y'' - \lambda y = 0, y(0) = y(1) = 0$

(2) $y'' + \lambda y = 0, y(0) = y(1) = 0$

(3) $y'' - \lambda y = 0, y(0) = 0, y(1) = 1$

(4) $y'' + \lambda y = 0, y(0) = 1, y(1) = 0$

17. Using the central difference schemes, the finite difference equation corresponding to the differential equation $y'' - 2y' + y = x^2$ at the grid x_i when $x_i - x_{i-1} = h$ is :

(1) $y_{i-1} - hy_i + y_{i+1} = x_i^2$

(2) $(1-h)y_{i-1} + (h^2 - 2)y_i + (1+h)y_{i+1} = h^2 x_i^2$

(3) $(1+h)y_{i-1} - (2-h^2)y_i + (1-h)y_{i+1} = h^2 x_i$

(4) $(1+h)y_{i-1} - (2-h^2)y_i + (1-h)y_{i+1} = h^2 x_i^2$

18. The partial differential equation $\sin^2 xU_{xx} + \sin 2xU_{xy} + \cos^2 xU_{yy} = x$ is :

(1) elliptic (2) parabolic (3) hyperbolic (4) circular

19. The solution of the partial differential equation $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ subject to $u(0, x, y, z) = e^z \sin x \cos y$ and $t \geq 0$ is :

(1) $u(t, x, y, z) = e^{z+t} \sin (x + t) \cos (y - t)$

(2) $u(t, x, y, z) = e^{z-t} \sin (x - t) \cos (y - t)$

(3) $u(t, x, y, z) = e^{z+t} \sin (x + t) \cos (y + t)$

(4) $u(t, x, y, z) = e^{z-t} \sin (x - t) \cos (y + t)$

20. Let $f : \mathrm{IR}^2 \to \mathrm{IR}$ be defined by $f(x) = x_1^2 - x_2^2$. Then :

(1) $[0,0]^T$ satisfies the first order necessary condition

(2) $[0,0]^T$ satisfies the second order necessary condition

(3) $[0,0]^T$ is a minimizer of f

(4) $[0,0]^T$ satisfies the second order sufficient condition

21. Consider the minimization of $f(x_1, x_2) = \frac{1}{2}x^T \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix} x - x^T \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $x \in \mathbb{R}^2$, using the conjugate direction method with $x^0 = [0, 0]^T$ and Q-conjugate directions $d^0 = [1, 0]^T$ and $d^1 = \left[-\frac{3}{8}, \frac{3}{4}\right]^T$, Then,

(1) $g^0 = [-1,1]^T$ (2) $\alpha_0 = \frac{1}{4}$

(3) $x^1 = \left[-\frac{1}{4}, 0\right]^T$ (4) $g^1 = \left[0, \frac{3}{2}\right]^T$

22. Let $f(x_1, x_2) = x_1^2 + \frac{1}{2}x_2^2 + 3$.. By taking $x^0 = [1,2]^T$, $H_0 = I_2$ and applying the rank one correction algorithm to minimize f, we get :

(1) $d^0 = [2,2]^T$ (2) $\alpha_0 = \frac{3}{2}$ (3) $\alpha_0 = \frac{2}{3}$ (4) $x^1 = \left[\frac{1}{3}, \frac{1}{3}\right]^T$

23. Total number of group homomorphisms from groups $\mathbb{Z}_{16} \to \mathbb{Z}_{24}$ are :

(1) 2 (2) 6 (3) 8 (4) 48

24. If W_1 and W_2 are subspaces of a finite dimensional vector space U, then annihilator $(W_1 \cap W_2)^0$ is equal to :

(1) $W_1^0 \cap W_2^0$ (2) $W_1^0 \cup W_2^0$ (3) $W_1^0 + W_2^0$ (4) $W_1 \ominus W_2$

25. In group U(24) = {1, 5, 7, 11, 13, 17, 19, 23} if H = {1, 13} and K = {1, 17}, then HK is given by :

(1) {1, 13, 1, 17} (2) {1, 13, 17} (3) {1, 5, 13, 17} (4) {1, 13, 17, 221}

26. Let U be the vector space of all m × m matrices over the field F and V be the vector space of all n × n matrices over the same field F. Then Hom (U, V), the vector space of all linear transformation from U to V is of dimension :

(1) $m+n$ (2) $(m+n)^2$ (3) mn (4) m^2n^2

27. Let $R = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| a_1, a_2, a_3, a_4 \in \mathbb{Z} \right\}$ and I be the ideal of R consisting of matrices with even integers. How many elements are in the quotient ring R/I ?

(1) 4 (2) 8 (3) 16 (4) infinite

28. Let X and Y be topological spaces and F : X → Y a continuous function. Then :

(1) if X is Housdorff space then Y is also Housdorff

(2) Both X and Y are Housdorff or none of them is Housdorff

(3) if Y is Housdorff then X is Housdorff

(4) Either X or Y is Housdorff space

29. Consider the following statements :

(A) The property of "compactness" is a hereditary property

(B) The property of "compactness" is a topological property

Then :

(1) (A) is true and (B) is false (2) (B) is true and (A) is false

(3) Both (A) and (B) are true (4) Both (A) and (B) are false

30. Let X be an uncountable set with cofinite topology. Then :

(1) X is first countable but not second countable.

(2) X is second countable.

(3) X is not first countable.

(4) X is separable.

31. Let $x_1 = \sqrt{2}$ and for any natural number n ≥ 1, $x_{n+1} = \sqrt{2 + x_n}$. Then :

(1) the sequence (x_n) is monotonically decreasing and $\lim\limits_{n \to \infty} x_n = 0$.

(2) the sequence (x_n) is monotonically increasing and $\lim\limits_{n \to \infty} x_n = \sqrt{2}$.

(3) the sequence (x_n) is not monotonically increasing.

(4) $\lim\limits_{x \to \infty} x_n = 2$.

32. Let f : IR → IR be a monotonic function and S denote the set of points where f is discontinuous. Then S is :

(1) a finite set

(2) a countable set

(3) a countably infinite set

(4) an uncountable set

33. Let E be a subset of IR. Then :

(1) if E is Lebesgue measurable then E is a Borel set

(2) if E is not a Borel set then E is Lebesgue measurable

(3) if E is a Borel set then E is Lebesgue measurable

(4) None of the above

34. Let $f(z) = \begin{cases} ze^{1/z} & z \neq 0 \\ 0 & z = 0 \end{cases}$ then z = 0 is a :

(1) pole of $f(z)$

(2) removable singular point of $f(z)$

(3) non-isolated singular point of $f(z)$

(4) essential singularity of $f(z)$

35. Let (X, || ||) be a normed linear space. Then the "norm" is :

(1) uniformly continuous function on X

(2) continuous on X but not uniformly continuous

(3) bounded function on X

(4) None of the above

36. By contour integration, the value of $\int_{-\infty}^{\infty} \frac{e^{x/2}}{1+e^{x}} dx$ is :

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) π (4) None of these

37. Let H be a Hilbert space over a field $\mathbb{C}$. If T_1 and T_2 are normal operators on H into itself such that either commutes with adjoint of the other, then :

(1) $T_1 + T_2$ is normal but $T_1 T_2$ is not normal

(2) $T_1 T_2$ is normal but $T_1 + T_2$ is not normal

(3) neither $T_1 + T_2$ is normal nor $T_1 T_2$ is normal

(4) $T_1 + T_2$ and $T_1 T_2$ both are normal

38. If (X, T_1) and (Y, T_2) are two topological spaces and $f : X \to Y$ is a homeomorphism on X onto Y, then f is :

(1) open but not closed (2) closed but not open

(3) neither closed nor open (4) closed as well as open

39. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n + x^{10}}$, $\forall$, $x \in$ IR is :

(1) uniformly as well as absolutely convergent

(2) uniformly but not absolutely convergent

(3) absolutely but not uniformly convergent

(4) neither absolutely nor uniformly convergent

40. Let (X, || ||) be a Banach space are $\mathbb{C}$. Then :

(1) every series in X is convergent

(2) every convergent series in X is absolutely convergent

(3) every absolutely convergent series in X is convergent

(4) every absolutely convergent series in X is not convergent

Attempt any five questions. Write answer in 150-200 words. Each question carries 16 marks. Answer each question on separate page, after writing Question Number.

1. If q_r and p_r, r = 1, 2, –, n are generalized coordinates and momenta variables respectively of a rigid body, then show that :

$$\sum_{r=1}^{n} p_r \dot{q}_r = 2T,$$

where T is the kinetic energy.

2. Derive Bernoulli's equation in its general form.

3. Use revised simplex method to minimize :

$6x_1 + 4x_2 + 7x_3 + 5x_4$,

Subject to : $x_1 + 2x_2 + x_3 + 2x_4 \leq 20$; $6x_1 + 5x_2 + 3x_3 + 2x_4 \leq 100$;

$$3x_1 + 4x_2 + 9x_3 + 12x_4 \leq 75; \quad x_1, x_2, x_3, x_4 \geq 0.$$

4. Let G be a finite abelian group and p be a prime such that p | O(G), then show that there exists an element a ∈ G such that $a^p = e$.

5. Show that no group of order 108 is simple.

6. Define a locally connected topological space. Give an example of a topological space which is connected but is not locally connected. Also, prove that every component of a locally connected space is open.

7. Define Cantor set. Show that Cantor set has Lebesgue measure zero. Is it countable ?

8. Prove that a normed linear space is a Banach space if and only if every absolutely summable series is summable. Using the above criterion, give an example of a normed linear space and show that it is not a Banach space.

9. Define the radius of convergence R of the power series $\sum_{n=0}^{\infty} a_n z^n$ and give one example. Show that :

$$\frac{1}{R} = \overline{\lim_{n \to \infty}} |a_n|^{1/n}$$

10. Solve the boundary value problem $y'' + xy' - y = 2x^2$, $y(0) = 0$, $y(1) = 1$, by using the Ritz's method and taking the approximating function as $y(x) = x + c_1 x(1 - x)$.

Roll No. : ....

Q. No. :

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FOR ROUGH WORK