F/o Science

RET/15/Test B

63

896	Statistics

Question Booklet No. 50

*(To be filled up by the candidate by **blue/black ball-point pen**)*

Roll No. | | | | | | | | | |

Roll No. (Write the digits in words) .....

Serial No. of OMR Answer Sheet .....

Day and Date ....

(Signature of Invigilator)

INSTRUCTIONS TO CANDIDATES

(Use only ***blue/black ball-point pen*** in the space above and on both sides of the ***Answer Sheet***)

1. *Within 10 minutes* of the issue of the Question Booklet, *Please ensure that you have got the correct booklet and it contains all the pages in correct sequence and no page/question is missing.* In case of faulty Question Booklet, bring it to the notice of the Superintendent/Invigilators immediately to obtain a fresh Question Booklet.
2. Do not bring any loose paper, written or blank, inside the Examination Hall *except the Admit Card without its envelope.*
3. *A separate Answer Sheet is given. It should not be folded or mutilated. A second Answer Sheet shall not be provided.*
4. Write your Roll Number and Serial Number of the Answer Sheet by pen in the space provided above.
5. *On the front page of the Answer Sheet, write by pen your Roll Number in the space provided at the top, and by darkening the circles at the bottom. Also, wherever applicable, write the Question Booklet Number and the Set Number in appropriate places.*
6. ***No overwriting is allowed in the entries of Roll No., Question Booklet No. and Set No. (if any) on OMR sheet and Roll No. and OMR sheet No. on the Question Booklet.***
7. *Any changes in the aforesaid-entries is to be verified by the invigilator, otherwise it will be **taken as unfair means.***
8. *This Booklet contains 40 multiple choice questions followed by 10 short answer questions. For each MCQ, you are to record the correct option on the Answer Sheet by darkening the appropriate circle in the corresponding row of the Answer Sheet, by pen as mentioned in the guidelines given on the first page of the Answer Sheet. For answering any five short Answer Questions use five Blank pages attached at the end of this Question Booklet.*
9. For each question, darken only one circle on the Answer Sheet. If you darken more than one circle or darken a circle partially, the answer will be treated as incorrect.
10. *Note that the answer once filled in ink cannot be changed. If you do not wish to attempt a* question, leave all the circles in the corresponding row blank (such question will be awarded zero marks).
11. For rough work, use the inner back page of the title cover and the blank page at the end of this Booklet.
12. Deposit ***both OMR Answer Sheet and Question Booklet*** at the end of the Test.
13. You are not permitted to leave the Examination Hall until the end of the Test.
14. If a candidate **attempts to use any** form of unfair means, he/she shall be liable to such punishment as the University may determine and impose on him/her.

Total No. of Printed Pages : 19

FOR ROUGH WORK

Research Entrance Test – 2015

No. of Questions : 50

Time : 2 Hours *Full Marks : 200*

Note : (i) This Question Booklet contains **40** Multiple Choice Questions followed by **10** Short Answer Questions.

(ii) Attempt as many MCQs as you can. Each MCQ carries **3 (Three)** marks. **1 (One)** mark will be deducted for each incorrect answer. **Zero** mark will be awarded for each unattempted question. If more than **one** alternative answers of MCQs seem to be approximate to the correct answer, choose the closest one.

(iii) Answer only **5** Short Answer Questions. Each question carries **16 (Sixteen)** marks and should be answered in **150-200** words. Blank **5 (Five)** pages attached with this booklet shall only be used for the purpose. Answer each question on separate page, after writing Question No.

1. Neoprene is polymer of :

   (1) Orlon (2) SAN (3) ABS (4) All of these

2. The reagent that can be used to distinguish between Glucose and Fructose is :

   (1) Bromine water (2) Fehling's solution

   (3) Tollen's reagent (4) Phenyl hydrazine

3. What will happen if a lysosome leaks inside the cell ?

   (1) The lysosomal enzymes will digest cell organelles

   (2) The lysosomal enzymes will become nonfunctional at pH 7.4 of the cytoplasm

   (3) The lysosomal enzymes will be secreted out of the cell

   (4) The leaked suicidal bag will make cell to commit suicide

4. Oxygen evolved during photosynthesis in plants comes from :

   (1) Splitting of water molecules

   (2) Breakdown of carbon dioxide

   (3) Carbohydrates accumulated by plants

   (4) Lipids

5. The contribution of Gregor Johann Mendel is related to the area of :

   (1) Plant classification (2) Genetics

   (3) Cell structure (4) Plant functions

6. Himalaya is :

   (1) Paleozoic tectonic mountain (2) Recent Folded mountain

   (3) Indian mountain (4) Eurasian mountain

7. A particle executes simple harmonic motion under the restoring force provided by a spring. The time period is T. If the spring is dived in two equal parts and one part is used to continue the simple harmonic motion, the time period will :

(1) remain T
(2) become 2T
(3) become T/2
(4) become $T/\sqrt{2}$

8. The efficiency of the Carnot's engine working between the steam point and the ice point is :

(1) 36.81%
(2) 26.81%
(3) 40%
(4) 16.8%

9. If $\vec{a} = 2i - 3j + 4k$ and $\vec{b} = 3i + 2j$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(1) 45°
(2) 90°
(3) 180°
(4) 120°

10. The value of the integral $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ is

(1) π
(2) $\frac{\pi}{2}$
(3) $\frac{\pi}{4}$
(4) $-\frac{\pi}{4}$

11. In the usual notations, the correct expression for $\sum_{11.2}$ is :

(1) $\sum_{11} - \sum_{12} \sum_{22}^{-1} \sum_{21}$
(2) $\sum_{11} - \sum_{21} \sum_{11}^{-1} \sum_{12}$
(3) $\sum_{22} - \sum_{12} \sum_{11}^{-1} \sum_{21}$
(4) $\sum_{22} - \sum_{21} \sum_{11}^{-1} \sum_{12}$

12. Let $X_{(1)} \leq X_{(2)} \leq \ldots\ldots \leq X_{(n)}$ be the order statistic of a random sample of size n from the uniform distribution $U(-0,0)$. then the statistic $T = (X_{(1)}, X_{(n)})$ is :

(1) Sufficient but not complete
(2) Both sufficient and complete
(3) Complete but not sufficient
(4) Neither sufficient nor complete

13. Let $Y_{(1)}, Y_{(2)}, \ldots\ldots\ldots\ldots, Y_{(n)}$, be the order statistic of a random sample from the probability density function $f(x) = \frac{1}{\pi\left[1+(y-0)^2\right]}$, $-\infty < y < \infty$

The minimal sufficient statistic is :

(1) $y_{(1)}$

(2) $\prod_{i=1}^{n} y_{(i)}$

(3) $\sum_{i=1}^{n} y_{(i)}$

(4) $(y_{(1)}, y_{(2)}, \ldots, y_{(n)})$

14. In the linear model $Y = X\beta + u$, if $E(uu') = G$ and best linear unbiased estimator of β is DY, then :

(1) $D = (X'X)^{-1}X'$

(2) $D = (X'GX)^{-1}X'$

(3) $D = (X'G^{-1}X)^{-1}X'G^{-1}$

(4) $D = (X'GX)^{-1}X'G$

15. In a 2^3 factorial experiment with r replicates, the block size is :

(1) 8 (2) r (3) 2r (4) 3r

16. In a BIBD with t treatments in b blocks of K plots each and r replicates, which one of the followings is NOT TRUE ?

(1) $rt = bk$ (2) $b \geq t$ (3) $r \geq k$ (4) $b \leq (r + t - k)$

17. Let $X = (X_1, X_2, X_3)'$ follows a three-variate normal distribution with mean vector O and the covariance matrix $\begin{bmatrix} 1 & 0 & 1/4 \\ 0 & 1/4 & 0 \\ 1/4 & 0 & \frac{1}{2} \end{bmatrix}$

Then $E[X_1 | X_2, X_3]$ is :

(1) $\frac{1}{2}(X_2 + X_3)$

(2) $\frac{1}{2}X_3$

(3) X_3

(4) $\frac{1}{4}X_2 + \frac{1}{2}X_3$

18. Let $x_1 = -2$, $x_2 = 1$, $x_3 = 3$, $x_4 = -4$ be the observed values of a random sample from the probability density function

$$f(x,\theta) = \frac{e^{-x}}{e^{-\theta} - e^{-\theta}}, \quad -\theta < x < \theta, \quad \theta > 0$$

Then the maximum likelihood estimator of θ is :

(1) 3 (2) 0.5

(3) 4 (4) 1.5

19. Let X and Y be independently and identically distributed random variables such that $P[X = k] = p_k > 0$ and $\sum_{k=0}^{\infty} p_k = 1$

If $P[X = t \mid (X + Y) = t] = P[X = (t-1) \mid (X + Y) = t]$, then X and Y follow :

(1) Poisson distribution (2) Negative binomial distribution

(3) Geometric distribution (4) None of these

20. Let $X_1, X_2, \ldots$ be independently and indentically distributed standard normal variables, then which one of the followings is TRUE ?

(1) $\frac{\sqrt{n}\,X_1}{\sqrt{\sum_{i=1}^{n} X_i^2}}$ has t-distribution with (n – 1) degrees of freedom

(2) $\frac{\sqrt{n}\,X_1}{\sqrt{\sum_{i=1}^{n} X_i^2}}$ has t- distribution with n degrres of freedom

(3) $\frac{\sqrt{n}\,X_1}{\sqrt{\sum_{i=2}^{n+1} X_i^2}}$ has t- distribution with (n – 1) degrees of freedom

(4) $\frac{\sqrt{n}\,X_1}{\sqrt{\sum_{i=2}^{n+1} X_i^2}}$ has t- distribution with n degrees of freedom

21. Let $\Omega = [0, 1]$ and S be the Borel field of subsets of Ω. Define X on Ω as follows :

$$X(w) = \begin{cases} w & \text{if} \quad 0 \le w \le \frac{1}{2} \\ w - \frac{1}{2} & \text{if} \quad \frac{1}{2} < w \le 1 \end{cases}$$

Let A and B belonging to S denote $\left(\frac{1}{4}, \frac{1}{2}\right)$ and $\left(\frac{3}{4}, 1\right)$ respectively. Further, C denotes $\left\{w : X(w) \in \left(\frac{1}{4}, \frac{1}{2}\right)\right\}$. Let S_1 : X is a random variable; $S_2 : C = A \cup B$ and $S_3 : C = A \cap B$. Then which of the following statements are always TRUE ?

(1) Only S_1 and S_2 (2) Only S_1 and S_3 (3) Only S_1 (4) Only S_2

22. If $X \ge 1$ is the critical region for testing $H_0 : 0 = 2$ against $H_1 : 0 = 1$ on the basis of a single observation from the population $f(x, 0) = 0e^{-0x}$, $x > 0$, then the value of type - I error is :

(1) $1/e^2$ (2) $(e - 1)/e^2$ (3) $(e^2 - 1/2$ (4) e^2

23. Let X be a random variable with probability density function

$$f(x, \mu) = \frac{1}{2} \exp(-|x - \mu|), -\infty < x < \infty$$

Then the maximum likelihood estimator of μ is :

(1) mean of the sample (2) median of the sample

(3) mode of the sample (4) mean and median of the sample

24. Let the random variable X follows N(0, 1) and the prior g(0) follow $N(\mu, 1)$. Then the posterior distribution of 0 based on single observation is :

(1) $N\left(\frac{\mu + x}{2}, \frac{1}{2}\right)$ (2) $N(\mu + x, 2)$

(3) $N\left(\mu, \frac{1}{2}\right)$ (4) $N\left(x, \frac{1}{2}\right)$

25. If 0 is the probability of success in binomial distribution, then the Jeffrey's prior for binomial sampling is :

(1) $g(0) \alpha \frac{1}{0}$ (2) $g(\theta) \alpha 0$

(3) $g(0) \alpha B\left(\frac{1}{2}, \frac{1}{2}\right)$ (4) $g(\theta) \alpha$ a constant

26. Let R(t) and h(t) respectively denote the reliability and hazard rate of a unit at time t, then which of the following relation is TRUE ?

(1) $\log R(t) = \int_0^t r(x)\,dx$ (2) $\log R(t) = -\int_0^t r(x)\,dx$

(3) $\log R(t) = 1 - \int_0^t r(x)\,dx$ (4) $\log R(t) = \exp\left\{-\int_0^t r(x)\,dx\right\}$

27. Which of the following distribution presents decreasing failure rate ?

(1) normal distribution

(2) gamma distribution with shape parameter greater than one

(3) log normal distribution

(4) none of these

28. Let X follow $N_p(0, \Sigma)$ and P is a $p \times p$ non-singular matrix such that PX follows $N_p(0, I)$. Then :

(1) P is identity matrix (2) P is an idempotent matrix

(3) $P'\Sigma P = I$ (4) $P\Sigma P' = I$

29. If for a finite population of size N, the probability of selection of the i^{th} unit in the sample of size n is $P_i (i = 1, 2, \ldots\ldots, N)$ and $P_i \alpha X_i$ where $\sum_{i=1}^{N} X_i = X$, then the usual ratio estimator becomes unbiased when the selection probability of a particular sample is :

(1) $\binom{N}{n} \frac{\sum_{i=1}^{n} X_i}{X}$ (2) $\binom{N-1}{n-1} \frac{\sum_{i=1}^{n} X_i}{X}$

(3) $\left[\binom{N-n}{n-1}\right]^{-1} \frac{\sum_{i=1}^{n} X_i}{X}$ (4) $\left[\binom{N}{n}\right]^{1} \frac{\sum_{i=1}^{n} X_i}{X}$

30. When sample size is large, the efficiency of the estimator of population mean in post stratification is approximately equal to that of :

(1) Neyman allocation
(2) Simple random sampling
(3) Cluster sampling
(4) Proportional allocation

31. If ρ represents the coefficient of correlation between two variables Y and X in a finite population, then the regression method is better than the ratio method of estimation for :

(1) all values of ρ
(2) negative values of ρ only
(3) for values of ρ lying between $\frac{1}{2}$ to 1 only
(4) for values of ρ lying between $-\frac{1}{2}$ to -1 only

32. Let $\{X_n, n > 0\}$ be a Markov chain with three states $[0, 1, 2]$ and with transition matrix

$$\begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$$

The initial probabilities are $P[X_0 = i] = \frac{1}{3}$, $i = 0, 1, 2$

Then $P[X_2 = 2, X_1 = 1, X_0 = 2]$ is equal to :

(1) 3/16
(2) 1/16
(3) 3/64
(4) 3/4

33. Let $X_1, X_2, \ldots\ldots X_n$ be a random sample from a Poisson distribution $P(\lambda)$. The Cramer-Rao Lower bound for the variance of an unbiased estimator of λ is :

(1) λ^2/n
(2) $\frac{\sqrt{\lambda}}{n}$
(3) $\frac{\lambda}{n}$
(4) $\frac{\lambda}{\sqrt{n}}$

34. For testing $H_0 : \theta = \frac{1}{2}$ against $H_1 : \theta = \frac{3}{4}$ on the basis of a single observation X from binomial distribution b(2, θ), a test function is defined as

$$\phi(x) = \begin{cases} 1 & if \quad x > 2 \\ \frac{1}{20} & if \quad x = 2 \\ 0 & if \quad x < 2 \end{cases}$$

The size of the test is :

(1) 1/80 (2) 1/20 (3) 1/60 (4) 1/10

35. In the following life table, some entries are missing :

x	$_5q_x$	l_x	$_5d_x$
20	0.006338	94864	
25	0.006650		
30	0.008087		
35		92879	

The values of l_{25} and l_{30} are respectively :

(1) (94564, 92140) (2) (94263, 93636)

(3) (94980, 92050) (4) (93500, 94564)

36. Given l_0 in a life table, the entries of other columns can be computed only with the knowledge of :

(1) age-specific fertility rates (2) crude death rate

(3) crude birth rate (4) age-specific mortality rates

37. Systematic sampling would be more efficient as compared with SRSWOR if :

(1) $\rho < -\frac{1}{nk-1}$ (2) $\rho > \frac{1}{(nk-1)}$

(3) $\rho < \frac{1}{(nk-1)}$ (4) $\rho = \frac{1}{(nk-1)}$

38. In $Y = X\beta + u$, $E(uu') = \sigma^2 I_n$, the value of $\hat{\beta}$ for which $(Y - X\hat{\beta})'(Y - X\hat{\beta})$ is minimum is b. Then $\frac{1}{\sigma^2}E(b-\beta)(b-\beta)'$ is :

(1) I_n (2) $X'X$ (3) $(X'X)^{-1}$ (4) $(XX')^{-1}$

39. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from a Bernoulli distribution with parameter p, $0 < p < 1$. For estimating p, the bias of the estimator $\frac{\sqrt{n} + 2\sum_{i=1}^{n} X_i}{2(n+\sqrt{n})}$ is :

(1) $\frac{1}{\sqrt{n}+1}\left(p - \frac{1}{2}\right)$ (2) $\frac{1}{\sqrt{n}+n}\left(\frac{1}{2} - p\right)$

(3) $\frac{1}{\sqrt{n}+1}\left(\frac{1}{2} + \frac{p}{\sqrt{n}}\right) - p$ (4) $\frac{1}{\sqrt{n}+1}\left(\frac{1}{2} - p\right)$

40. For the following transportation problem, one applies the North-West corner rule to find the first feasible solution :

	D_1	D_2	D_3	D_4	Availability
O_1	5	3	6	2	19
O_2	4	7	9	1	37
O_3	3	4	7	5	34
Demand	16	18	31	25	

The value of the objective function would be :

(1) 586 (2) 517 (3) 580 (4) 590

Attempt any five questions. Write answer in 150-200 words. Each question carries 16 marks. Answer each question on separate page, after writing Question Number.

1. What is a parallel system ? Obtain the expressions for the reliability, hazard rate and mean time to system failure of a system of n components assuming constant hazard rate for each component.

2. Define consistency and state the invariance property of consistency.

3. What is conjugate prior ? Give examples of conjugate priors.

4. Describe the method of finding the discontinuity points of cumulative density function of a random variable whose characteristic function is $(q + pe^{it})$.

5. The joint probability density function of X and Y is given by

$$f(x, y) = \frac{1}{2x^2y}, 1 \le x \le \infty; \frac{1}{x} < y < x$$

Find the conditional distribution of X given Y.

6. In order to resolve the problem of non-response explain the method of sub-sampling of non-respondents and define an unbiased estimator for population mean under this method.

7. Solve the following 2-machine and 7-job sequencing problem :

job :	1	2	3	4	5	6	7
Machine A :	10	12	13	7	14	5	16
Machine B :	15	11	8	9	6	7	16

8. Discuss the direct method of standardization of death rates with an example.

9. Define Mahalanobis D^2-statistic and explain its uses. How the T^2-statistic is related with D^2-statistics ?

10. Define Midzuno method of sampling and hence obtain the expressions of π_i and π_{ij}.

Q. No. :

Roll No. : .....

Q. No. :

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FOR ROUGH WORK